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 We used "z" to represent the number of children aged <6 years with BLLs of 5–9 µg/dL in a county i re event, we assumed "z" follows a Poisson

distribution with parameter " " and "m" as the number of children age <6 years who were tested

for BLL (equation 1):

p(z/) = e

$^{-(m\ .\ )}$(m .)^{z}/z! (1)

where is the rate of children with BLLs of 5–9 µg/dL

i.e., = (children with BLLs of 5–9 µg/dL) / (children tested for BLL)

If follows a gamma (,) prior

i.e., p( ) = e $^{-(\ )}$ $^{-1}$/ ( ) (2)

where > 0, then posterior distribution of is given by

p(/z) = p(z/) X p( )/p(z)

i.e., p(ōPÈ

If we assume that the prior information about parameter , the rate of children with BLLs of 5-9 µg/dL, can be obtained from a small group of counties in Georgia, who we believe has the same rate ( ) of 5-9 µg/dL BLLs among children aged <6 years, then the posterior for   can be estimated from equation (3).

We suppose z_j is the number of children aged <6 years with BLLs of 5–9 µg/dL among x_j children from county "j". Then assuming it follows a Poisson distribution, we have the following:

$p(z_j/ \;) = e^{-(x_j \;)} (x_j \;)^{z_j} / z_j!$ (4)

where   is the same as defined earlier.

So, the likelihood function for n counties with the same parameter   is given as follows:

$L(\; z_j / \;) = e^{-(\; x_j \;)} \; (x_j \;)^{z_j} / z_1! \, z_2! \ldots z_n!$ (5)

So,

$L(\; z_j / \;) \quad e^{-(\; x_j \;)} (\;)^{\; z_j}$ (6)

If for all these n counties we assume that   follows a non-informative prior 1/ , i.e., p( ) = 1/ , then from equation (6), the posterior distribution of   is given by the following:

$p(\; / \; z_j) \quad e^{-(\; x_j \;)} (\;)^{\; z_j} . 1/$

i.e., $p(\; / \; z_j) \quad e^{-(\; x_j \;)} (\;)^{\; z_j - 1}$ (7)

This is a gamma ($_2$, $_2$), where

$_2 = \; z_j$ and $_2$

county. We then can use known and from equation (8) in equations (2) and (3) to evaluate the prior and posterior distributions of the parameter in the targeted county.

The joint distribution of data z and the parameter are given by the following:

$p(z, \ ) = p(\ ) \times p(z/ \ )$, and also

$p(z, \ ) = p(z) \times p(\ /z)$

Thus, $p(z) \times p(\ /z) = p(\ ) \times p(z/ \ )$, giving

$$p(z) = p(\ ) \times p(z/ \ )/p(\ /z) \qquad (9)$$

Here, $p(\ )$ and $p(\ /z)$ are the known prior and posterior distributions, respectively, of the parameter . Thus, p(